THE FLOW OF A DIELECTRIC LIQUID

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The nature of the forces acting on a dielectric liquid in an electric field in the presence of a temperature gradient is investigated. It is shown that the electrical forces produced by nonuniformity in the electrical conductivity of the medium can give rise to a hydrodynamic flow.

A number of experiments [1-5] have studied the behavior of a dielectric liquid in an electrostatic field between horizontal plane-parallel electrodes in the presence of a temperature gradient $\partial T/\partial z$ (the z axis is directed upwards). It has been established that a downward hydrodynamic flow is observed in the column of liquid between the electrodes whatever the polarity of the field. There is no common explanation for the physical nature of this electrohydrodynamic effect. Attempts to explain the experimental data by the movement of charged particles and by a nonuniformity in the dielectric properties of the medium [4-6] have not met with sucess.

In this paper we consider the effect of the temperature gradient $k = \partial T/\partial z = \text{const} > 0$ ($\partial T/\partial x = \partial T/\partial y = 0$) on the nonuniformity in the electric field E(z) and we show that the electric force caused by this nonuniformity is in agreement with the experimental data as regards its absolute value and direction.

For $k \neq 0$, the electrical conductivity of a liquid dielectric over a small temperature range (~ 20°C) can be described by the exponential relationship [7]

$$\sigma = \sigma_0 \exp\{\alpha (T - T_0)\} = \sigma_0 \exp\{\alpha k z\} \tag{1}$$

where σ_0 is the conductivity at temperature T_0 and α is a constant which characterizes the $\sigma(T)$ relationship over the given range. When steady-state spatial currents are set up in a medium with nonuniform conductivity, surface charges are produced [3-6] and these lead to a nonuniformity in the electric field. Using the continuity equation for a steady current

$$\operatorname{div} \sigma E = 0 \tag{2}$$

and expression (1), we find that

$$E(z) = E_0 \exp\{-\alpha (T - T_0)\} = E_0 \exp\{-\alpha k z\} \tag{3}$$

where E_0 is the field strength at T_0. If we neglect the dielectric nonuniformity $\partial\varepsilon/\partial T$, we get the electric force density acting on the dielectric medium as [8]

$$F = [(\varepsilon - 1)/8\pi] \nabla E^2 \tag{4}$$

Substituting (3) into this expression, we get

$$F(z) = -[(\varepsilon - 1)/4\pi] \alpha k E_0^2 \exp\{-2\alpha k z\} \tag{5}$$

It follows from (5) that the force F is directed towards lower temperatures, i.e., the direction corresponds to the observed hydrodynamic flow. Since the electric conductivity of liquid dielectrics depends

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strongly on the temperature (for transformer oil $\alpha \sim 0.06$ deg^{-1}), the absolute value of F exceeds the forces produced by nonuniformity in the dielectric properties [4].

It is difficult to establish an exact connection between the value of F and the intensity of the liquid flow observed experimentally because the hydrodynamic flow process usually upsets the stationarity conditions for the temperature field and electric current which were used in the derivation of (5). It is, however, possible to make some estimates on the basis of the results from [2]. The experiment was carried out as follows. The upper electrode was heated for a time $\tau_0 \sim 10$ sec. An electric field of $E \sim 15$ kV/cm was then applied and the appearance of heated liquid at the lower electrode was noted after a time $\tau = L/v_* \ll L^2/a$. Here L = 1 cm is the distance between the electrodes and a is the coefficient of thermal conductivity for the liquid (transformer oil). The experimental value of the velocity $v_* \sim 10$ cm/sec.

The heating of the upper electrode produces a temperature gradient in the heated layer of liquid of $k \sim (T-T_0)/l$ ($T-T_0 \sim 20$°C, $l \sim \sqrt{a\tau_0} \sim 0.1$ cm). The force acting on a column of liquid between the electrodes when the liquid begins to move can be estimated as

$$\pi R^2 \int_0^l F(z)\,dz \tag{6}$$

where R is the radius of the electrode. The force (6) acts in the high-temperature region. No nonuniformity in the field can be established in the unheated part of the liquid column because the relaxation time $\varepsilon/4\pi\sigma \gg \tau \sim 0.1$ sec.

As the boundary of the heated liquid moves, the integrated value (6) changes only slightly because the decrease in the gradient k is compensated by the increase in the upper limit (region where the force acts). Thus the force acting on the liquid column can be taken as more or less constant.

If we assume that the heated liquid reaches the lower electrode under the action of the force (6), then by substituting the numerical values of the parameters used in [2], we arrive at the estimate of $v_* \sim 30$ cm/sec. This is in order-of-magnitude agreement with the experiment and shows that the electric force F caused by nonuniformity of the electrical conductivity might play the important role in this electrohydrodynamic phenomenon.

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